CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4037/02

Paper 2

October/November 2003

2 hours

Additional Materials: Answer Booklet/Paper
Graph paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **6** printed pages and **2** blank pages.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate
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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\, r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

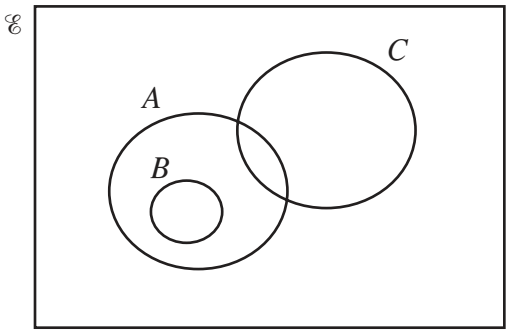
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc \cos A.$$
$$\Delta = \tfrac{1}{2} bc \sin A.$$

1 The line $4y = x + 11$ intersects the curve $y^2 = 2x + 7$ at the points A and B. Find the coordinates of the mid-point of the line AB. [4]

2 Show that $\cos\theta\left(\frac{1}{1-\sin\theta} - \frac{1}{1+\sin\theta}\right)$ can be written in the form $k\tan\theta$ and find the value of k. [4]

3 Solve the equation $\log_2 x - \log_4(x-4) = 2$. [4]

4

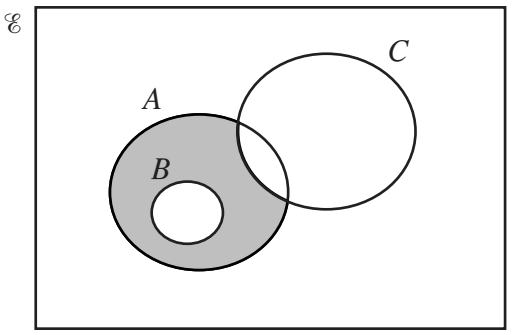


The diagram shows a universal set $\mathscr{E}$ and the three sets A, B and C.

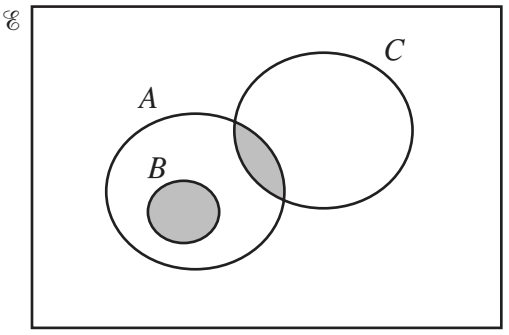
(i) Copy the above diagram and shade the region representing $(A \cup C) \cap B'$.

For each of the diagrams below, express, in set notation, the set represented by the shaded area in terms of A, B and C.

(ii)



(iii)



[4]

5 Obtain

(i) the first 3 terms in the expansion, in descending powers of x, of $(3x - 1)^5$, [3]

(ii) the coefficient of x^4 in the expansion of $(3x - 1)^5(2x + 1)$. [2]

6 A particle travels in a straight line so that, t s after passing a fixed point A, its speed, v ms^{-1}, is given by

$$v = 40(e^{-t} - 0.1).$$

The particle comes to instantaneous rest at B. Calculate the distance AB. [6]

7 Given $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix}$, write down the inverse of **A** and of **B**. [3]

Hence find

(i) the matrix **C** such that $2\mathbf{A}^{-1} + \mathbf{C} = \mathbf{B}$, [2]

(ii) the matrix **D** such that $\mathbf{BD} = \mathbf{A}$. [2]

8 A garden centre sells 10 different varieties of rose bush. A gardener wishes to buy 6 rose bushes, all of different varieties.

(i) Calculate the number of ways she can make her selection. [2]

Of the 10 varieties, 3 are pink, 5 are red and 2 are yellow. Calculate the number of ways in which her selection of 6 rose bushes could contain

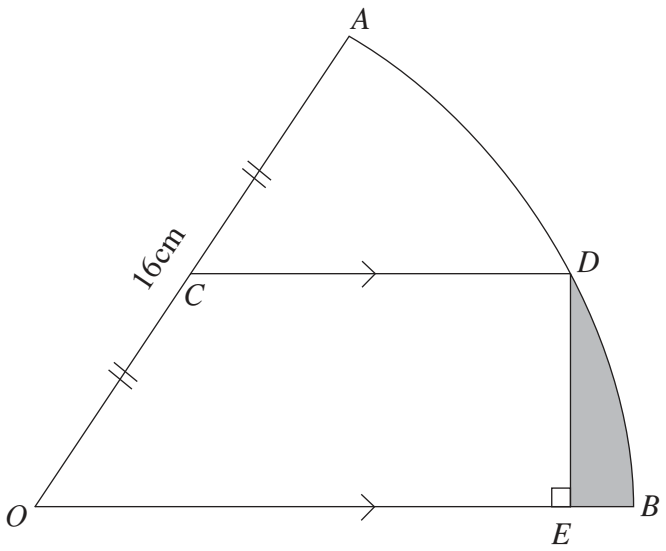
(ii) no pink rose bush, [1]

(iii) at least one rose bush of each colour. [4]

9 **(i)** Given that $y = (2x + 3)\sqrt{4x - 3}$, show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{kx}{\sqrt{4x - 3}}$ and state the value of k. [5]

(ii) Hence evaluate $\displaystyle\int_1^7 \frac{x}{\sqrt{4x - 3}}\, dx$. [3]

10



In the diagram, OAB is a sector of a circle, centre O and radius 16 cm, and the length of the arc AB is 19.2 cm. The mid-point of OA is C and the line through C parallel to OB meets the arc AB at D. The perpendicular from D to OB meets OB at E.

(i) Find angle AOB in radians. [2]

(ii) Find the length of DE. [2]

(iii) Show that angle DOE is approximately 0.485 radians. [2]

(iv) Find the area of the shaded region. [4]

11 A particle, moving in a certain medium with speed v ms^{-1}, experiences a resistance to motion of R N. It is believed that R and v are related by the equation $R = kv^{\beta}$, where k and β are constants.
The table shows experimental values of the variables v and R.

v	5	10	15	20	25
R	32	96	180	290	410

(i) Using graph paper, plot lg R against lg v and draw a straight line graph. [3]

Use your graph to estimate

(ii) the value of k and of β, [5]

(iii) the speed for which the resistance is 75 N. [2]

12 Answer only **one** of the following two alternatives.

EITHER

Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f}: x \mapsto 3x - 2, \quad x \neq \tfrac{4}{3},$$
$$\mathrm{g}: x \mapsto \frac{4}{2 - x}, \quad x \neq 2.$$

(i) Solve the equation $\mathrm{gf}(x) = 2$. [3]

(ii) Determine the number of real roots of the equation $\mathrm{f}(x) = \mathrm{g}(x)$. [2]

(iii) Express f^{-1} and g^{-1} in terms of x. [3]

(iv) Sketch, on a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, stating the coordinates of the point of intersection of the two graphs. [3]

OR

(i) Find the value of a and of b for which $1 - x^2 + 6x$ can be expressed in the form $a - (x + b)^2$. [3]

A function f is defined by $\mathrm{f}: x \mapsto 1 - x^2 + 6x$ for the domain $x \geqslant 4$.

(ii) Explain why f has an inverse. [2]

(iii) Find an expression for f^{-1} in terms of x. [2]

A function g is defined by $\mathrm{g}: x \mapsto 1 - x^2 + 6x$ for the domain $2 \leqslant x \leqslant 7$.

(iv) Find the range of g. [2]

(v) Sketch the graph of $y = |\mathrm{g}(x)|$ for $2 \leqslant x \leqslant 7$. [2]

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